**International Journal of Advanced Research in**

**Mechanical and Production Engineering and Development**

Volume: 1 Issue: 1 08-Jun-2013,ISSN_NO: 2320-7256

# Elasticity Based Stress Solution of Curved Bar under Pure Bending and its Numerical Validation

Suhas Ankalkhope, Nilesh Jadhav, Prof.Dr.Sunil Bhat.

School Of Mechanical and Building Sciences,

VIT University, Vellore - 632014, Tamil Nadu, India

**nilesh_jadhav@yahoo.com: +91-9790543697**

## Abstract

In this paper, stress in curved bar under pure bending is estimated from the principles of elasticity. It is the case of axi-symmetry. Tangential and radial stress distribution is found analytically using Airy‟s stress function in polar coordinates and with the help of necessary boundary conditions. The curved bar is numerically modelled by finite element analysis. Variation of stress values with radius is obtained. Theoretical and numerical results are in excellent agreement with each other.

## Nomenclature

$\sigma_\theta$ - Tangential Stress (circumferencial stress) ar-Radial Stress

Trø-Shear stress

a- Inner radius b-Outer radius r-Mean radius

M-Moment applied E-Modulus of elasticity

## 1. Introduction

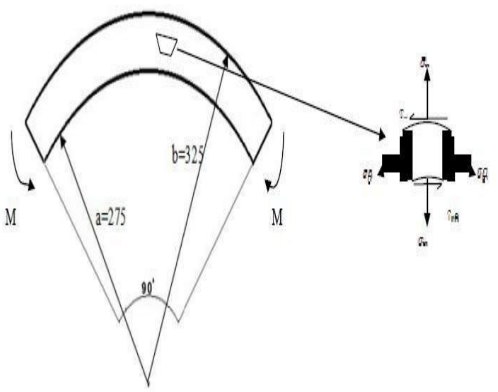
Sufficient attention is not given to boundary conditions in conventional theories of stress solution. On the other hand, principles of elasticity use stress function, given by „Airy‟, that is made to satisfy boundary conditions resulting in more accurate stress solutions. Elasticity based solutions find lot of application in higher research.

## 2. Aim

To use principles of elasticity in obtaining stress solution of curved elastic beam subjected to pure bending and then validate the solution by finite element method.

## 3. Problem Definition

Refer Fig. 1. A steel curved bar (included angle = 90o) of square cross section (50 mm x 50 mm) with mean radius of 300mm is subjected to a Moment of 50 N-m at its ends. (a = 275mm and b = 325mm.). Steel has E value of 210GPa and Poisons ratio of 0.33.



**4. Analysis-**

**A. Analytical Solution (Theoretical) -**

shear transmitted across all and hence the stress field is independent of . So it is an Axi- forces are symmetric problem and we therefore invoke a solution using the following Airy‟s stress function defined for Axi-symmetric case:-

$$\phi = A + B\log_e r + Cr^2 + Dr^2\log_e r \qquad \text{..........................(1)}$$

Where elastic constants A, B, C, D are obtained from the boundary conditions.

Radial normal stress, tangential normal stress and shear stress components are given by,

$$\sigma_r = \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}; \quad \sigma_\theta = \frac{\partial^2\phi}{\partial r^2} \text{ and } \quad \tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right)$$

Which for an Axi-symmetric case reduce to

$$\sigma_r = \frac{1}{r}\frac{\partial\phi}{\partial r}, \quad \sigma_\theta = \frac{\partial^2\phi}{\partial r^2}, \quad \tau_{r\theta} = 0 \qquad \text{.................................(2)}$$

From eq. (1) and (2)

$$\sigma_r = \frac{B}{r^2} + 2C + D(1 + 2\log_e(r)) \text{ and}$$

$$\sigma_\theta = -D/r^2 + 2C + D(3 + 2\log_e(r)) \quad \text{.........}$$

Boundary conditions for the problem

(1) $\sigma_r = 0 \quad$ at $r = a, b$

2) $M = \int_a^b \sigma_\theta . r\, dr \quad$ at $\theta = 0$

and an additional equation is obtained by

$$M = B\left(\log_e\left(\frac{a}{b}\right)\right) + (C + D)(b^2 \;\; a^2) + D(b^2\log_e b \;\; a^2\log_e(a)) \qquad \text{...............(6)}$$

Solving eq (4,5,6)

Expressions for elastic constant are found to be as,

$$B = \frac{\frac{M}{G}\left(\ln\left(\frac{a^2}{b^2}\right)a^2b^2\right)}{a^2 - b^2}$$

$$C = -\frac{M}{2G\left(1 + 2\ln(a) - \frac{\frac{M}{G}\left(\ln\left(\frac{a^2}{b^2}\right)b^2\right)}{2(a^2 - b^2)}\right)}$$

$$D = \frac{M}{G}$$

Where $G = \dfrac{b^2\ln\dfrac{a^2}{b^2\left(2a^2\ln\left(\frac{a}{b}\right) + a^2 - b^2\right)}}{2(a^2 - b^2)} + \dfrac{b^2 - a^2 + 2b^2\ln\frac{b}{a}}{2}$

But we have from fig (1) as a=275mm, b=325mm, M=50Nm

Therefore we get values of elastic constants as,

B=32055466.95 C= -2416.012064 D=360.3331

After this, value of $\sigma_\theta$ - at r = a and r = b are calculated. They are found compressive at r = a

tensile at r = b.

i.e.

$$\sigma_\theta\,(\text{at } r = a) = -127.08168 \text{ N/}$$

$$\sigma_\theta\,(\text{at } r = b) = 113.6987768 \text{ N/mm}^2$$

Position of neutral axis is located by equating to zero. It is found that neutral axis is located at

r=299.30519 mm.

For determination of tangential and radial stress distribution in a curved bar values of and are calculated at different „r‟.

**B. Numerical Solution-**

Numerical solution is estimated by using Finite Element Analysis (FEA). Mesh model of the bar in

Ansys is shown in Fig. 2.

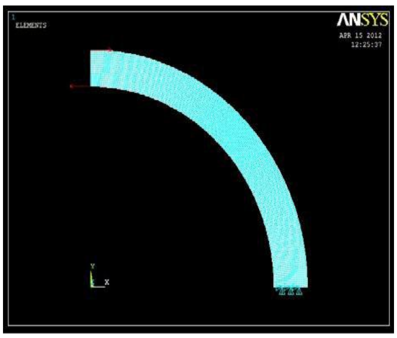



Fig.2 Mesh model with forces and constraints

Discipline - Structural

Analysis type - Static

Type of Element - PLANE 183

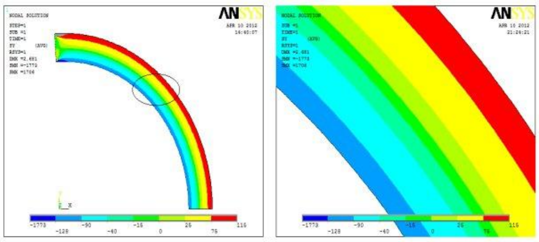
Number of nodes - 12751

Number of Elements - 4100

**5. Results-**

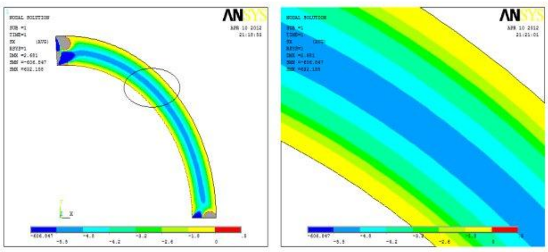
Nodal solutions are displayed in Fig.3.Analytical and numerical values are provided in Table1. Comparison between the values is provided in Fig. 4 and Fig. 5.

1. Tangential stress $(\sigma_{\theta})$ -



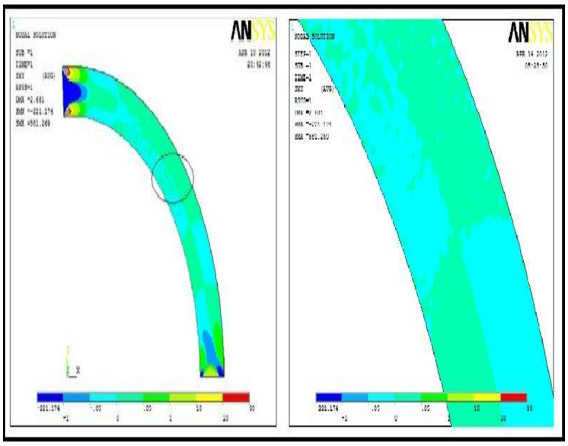
VIEW AT A

2. Radial Stress $(\sigma_r)$-



VIEW AT B

3. Shear Stress $(\tau_{r\theta})$ -



VIEW AT C

**Fig. 3 Stress solutions obtained from FEA**

**TABLE 1**

| SR.NO. | RADIUS (MM) | $\sigma_\theta$(ANALYTICAL) (N/mm2) | $\sigma_\theta$(NUMERICAL) (N/mm2) | $\sigma_r$(ANALYTICAL) (N/mm2) | $\sigma_r$(NUMERICAL) (N/mm2) |
|---|---|---|---|---|---|
| 1 | 275 | -127.0816879 | -127.061 | 0 | -0.011097 |
| 2 | 280 | -99.0931688 | -99.0734 | -2.01786497 | -2.02843 |
| 3 | 285 | -72.11718476 | -72.0983 | -3.482875067 | -3.49278 |
| 4 | 290 | -46.09222992 | -46.074 | -4.440547642 | -4.44986 |
| 5 | 295 | -20.96170449 | -20.9444 | -4.93231241 | -4.94109 |
| 6 | 300 | 3.326558637 | 3.343 | -4.995931363 | -5.00423 |
| 7 | 305 | 26.82071421 | 26.8367 | -4.665868484 | -4.67373 |
| 8 | 310 | 49.56527872 | 49.5807 | -3.973617425 | -3.9811 |
| 9 | 315 | 71.60146396 | 71.6164 | -2.947992272 | -2.95512 |
| 10 | 320 | 92.96747299 | 92.982 | -1.615386148 | -1.62221 |
| 11 | 325 | 113.6987768 | 113.713 | 0 | -0.00662853 |

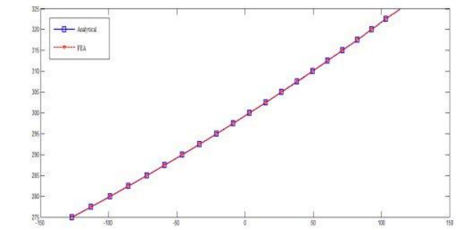


**Fig.4 Analytical and FEA tangential stress Vs radius**

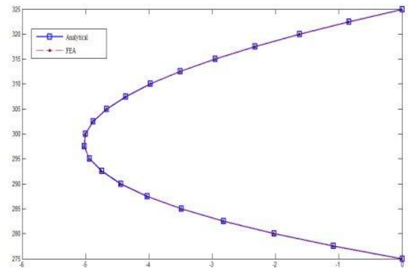


**Fig.5 Analytical and FEA Radial stress Vs radius**

**6. Discussion-**

An analytical solution procedure is used for circular bar of rectangular cross section subjected to bending moment at the ends. The bending moment is constant along cross section of bar. This is the case of axi- symmetry. So the stress distribution is independent of □ and is same at all the cross sections. Tangential stress is zero at mean radius and varies linearly along radius. At inner radius, it is negative (compressive) and is positive (tensile) at outer radius. The values are maximum at the ends. The radial stress is compressive and varies non-linearly along radius. It is maximum at the centre and zero at the ends. Shear stress is zero throughout due to axi-symmetry. The analytical results are compared with the FEA solution. The values are in excellent agreement with each other thereby validating the work.

**Conclusion-**

1. Tangential Stress is compressive at inner fibre and tensile at outer fibre

2. Tangential stress $(\sigma_\theta)$ is zero at neutral axis (At r=299.3051)

3. Radial stress is compressive, maximum at the centre and zero at ends

4. Shear stress is zero throughout

5. Theoretical and FEA results are in excellent agreement with each other

**References-**

1. “Solid mechanics and its applications” by J.R. Barber, University of Michigan, Ann Arbor, U.S.A. 2nd Edition.

2. Fracture Mechanics by Nestor Perez, University of Puerto Rico, 2004 Kluwer Academic Publishers.

3. Applied Elasticity by T.G.Sitharam and L Govinda Raju, Interline Publishing.

4. Principle of solid Mechanics by Rowland Richards, Jr., CRC Press.

5. Mechanics of material by Dr. Ibrahim A Assakkaf, University of Maryland, College Park, 2003.

6. BENDING OF MULTILAYERED CURVED BARS by K. H. Lo & H. D. CONWAY.