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STEADY MOTION OF VISCOUS LIQUID DUE TO A SLOWLY ROTATING SPHERE IN MAGNETIC FIELD

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ABSTRACT– *In this paper we have investigated the steady motion of viscous liquid due to a slowly rotating sphere in magnetic field. We have investigated the angular velocity, couple and rate of dissipation of energy.*

KEY WORDS: Steady poiseuille flow, viscous parallel plates, incompressible fluid and magnetic field.

NOMENCLATURE:

u =Velocity component along x–axis

v = Velocity component along y–axis

w= Velocity component along z–axis

t = the time

ρ = The density of fluid

P = the fluid pressure

K= the thermal conductivity of the fluid

μ = Coefficient of viscosity

υ = Kinematic viscosity

Q= the volumetric flow

r = Radius of sphere

N=Couple

INTRODUCTION:

We have investigated the steady motion of viscous liquid due to a slowly rotating sphere in magnetic field. Attempts have been made by several researchers i.e. J. M. Hamilton, J. Kim & F. Waleffe [1] Regeneration mechanisms of near-wall turbulence structures. T. Havarneanu, C. Popa, & S. S. Sritharan [2] Exact Internal Controllability for Magneto-Hydrodynamic Equations in Multi-connected Domains. R. D. Henderson & G. E. Karniadakis [3] unstructured spectral element methods for simulation of turbulent flows. H. H. Stapelberg & D. Mewes [4] the pressure loss and slug frequency of liquid-gas slug flow in horizontal pipes. H. Herwig & G. Wicken [5] the effect of variable properties on laminar boundary layer flow. S. Hou, Q. Zou, S. Chen, G. Doolen & A. C. Cogley [6] simulation of cavity flow by the lattice Boltzmann method. H. Huang & B. R. Wetton [7] discrete compatibility in finite difference methods for viscous incompressible fluid flow. O. A. Hurricane, B. H. Fong & S. C. Cowley [8] nonlinear magneto hydrodynamic detonation. G. J. Hwang & J. P. Sheu [9] Liquid solidification in combined hydrodynamic and thermal entrance region of a circular tube. J. Y. Jang & J. L. Chen [10] forced convection in a parallel plate channel partially filled with a high porosity medium. P. Janhari & N. H. harhimi [11] on the surfacial sediments of the fresh. In this paper we have investigated the angular velocity, Couple on the sphere and rate of dissipation of energy.

FORMULATION OF THE PROBLEM:

$$Let\ \bar{q} = q(u,v,w) \qquad Where\ \ w = 0, \quad u = -\omega y \quad \& \quad v = \omega x ....(1)$$

$$So\ that\ \ \frac{\partial u}{\partial x} = 0\ \ \frac{\partial v}{\partial y} = 0\ \&\ \frac{\partial w}{\partial z} = 0 \qquad \Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$\therefore$ The equation of continuity is satisfied where ω is the angular velocity s.t $\omega = \omega(r)$ *Where* $r^2 = x^2 + y^2 + z^2$

Navier – stokes equation in the absence of body Forces

$$\frac{d\overline{q}}{dt}=\frac{\partial\overline{q}}{\partial t}+\left(\overline{q}.\nabla\right)\overline{q}=-\frac{1}{\rho}\nabla p+\upsilon\nabla^{2}\overline{q}+\frac{\sigma B_{0}^{2}}{\mu}\upsilon\overline{q}$$

Since motion is steady $\Rightarrow \frac{\partial\overline{q}}{\partial t}=0$ but $\overline{q}$ very small and hence neglecting square of velocities

$\left(\overline{q}.\nabla\right)\overline{q}=0$ With these values (i) becomes $-\frac{1}{\rho}\nabla p+\frac{\mu}{\rho}\nabla^{2}\overline{q}+\frac{\sigma B_{0}^{2}}{\rho\mu}\mu\overline{q}=0$

$$\Rightarrow \mu\left\{\nabla^{2}\overline{q}+\frac{\sigma B_{0}^{2}}{\mu}\overline{q}\right\}=\nabla p$$

But $\nabla^{2}q^{2}=\nabla^{2}u\hat{i}+\nabla^{2}v\hat{j}+\nabla^{2}w\hat{k}$

$$\mu(\nabla^{2}ui+\nabla^{2}vj)+\frac{\sigma B_{0}^{2}}{\mu}\mu\left(ui+vj\right)=\frac{\partial p}{\partial x}i+\frac{\partial p}{\partial y}j+\frac{\partial p}{\partial z}k$$

Comparing $\mu\left(\nabla^{2}u+\frac{\sigma B_{0}^{2}}{\mu}u\right)=\frac{\partial p}{\partial x}.............(2)$

$$\mu\left(\nabla^{2}v+\frac{\sigma B_{0}^{2}}{\mu}v\right)=\frac{\partial p}{\partial y}...........(3)$$

and $\frac{\partial p}{\partial z}=0...........(4)$

SOLUTION OF THE PROBLEM:

$$u=-\omega y\Rightarrow\frac{\partial^{2}u}{\partial x^{2}}=-y\frac{\partial^{2}\omega}{\partial x^{2}},\qquad\frac{\partial^{2}u}{\partial z^{2}}=-y\frac{\partial^{2}\omega}{\partial z^{2}}$$

$$\frac{\partial u}{\partial y}=-\omega-y\frac{\partial\omega}{\partial y}\qquad\Rightarrow\frac{\partial^{2}u}{\partial y^{2}}=-\frac{\partial\omega}{\partial y}-\frac{\partial\omega}{\partial y}-y\frac{\partial^{2}\omega}{\partial y^{2}}$$

$$\therefore\nabla^{2}u=\frac{\partial^{2}u}{\partial x^{2}}+\frac{\partial^{2}u}{\partial y^{2}}+\frac{\partial^{2}u}{\partial z^{2}}=-y\frac{\partial^{2}\omega}{\partial x^{2}}-2y\frac{\partial\omega}{\partial y}-y\frac{\partial^{2}\omega}{\partial y^{2}}-y\frac{\partial^{2}\omega}{\partial z^{2}}=-y\nabla^{2}\omega-2\frac{\partial\omega}{\partial y}=-y\left[\nabla^{2}\omega+\frac{2}{y}\frac{\partial\omega}{\partial y}\right]$$

Similarly $\nabla^{2}v=x\left\{\nabla^{2}\omega+\frac{2}{x}\frac{\partial\omega}{\partial x}\right\}$ *But* $r^{2}=x^{2}+y^{2}+z^{2}$

$$\frac{\partial r}{\partial x}=\frac{x}{r}\ ,\ \frac{\partial r}{\partial y}=\frac{y}{r}\ \&\ \frac{\partial r}{\partial z}=\frac{z}{r}\ \&\ \frac{\partial \omega}{\partial x}=\frac{\partial \omega}{\partial r}.\frac{\partial r}{\partial x}=\frac{x}{r}\frac{\partial \omega}{\partial r}$$

$$\frac{\partial^2 \omega}{\partial x^2}=\frac{1}{r}\frac{\partial \omega}{\partial r}+\frac{x}{r}\left[\frac{\partial^2 \omega}{\partial r^2}\right]\frac{\partial r}{\partial x}+\left(-\frac{x}{r^2}\right)\left(\frac{x}{r}\right)\frac{\partial \omega}{\partial r}\quad=\frac{1}{r}\frac{\partial \omega}{\partial r}+\frac{x^2}{r^2}\frac{\partial^2 \omega}{\partial r^2}-\frac{x^2}{r^3}\frac{\partial \omega}{\partial r}$$

$$\Sigma\frac{\partial^2 \omega}{\partial x^2}=\frac{1}{r}\frac{\partial \omega}{\partial r}\ \Sigma 1-\frac{1}{r^3}\frac{\partial \omega}{\partial r}\ \Sigma x^2+\frac{1}{r^2}\frac{\partial^2 \omega}{\partial r^2}\Sigma x^2$$

$$\nabla^2\omega=\frac{3}{r}\frac{\partial \omega}{\partial r}-\frac{1}{r}\frac{\partial \omega}{\partial r}+\frac{\partial^2 \omega}{\partial r^2}\quad=\frac{2}{r}\frac{d\omega}{dr}+\frac{d^2\omega}{dr^2}$$

$$Also\quad \frac{2}{x}\frac{\partial \omega}{\partial x}=\frac{2}{r}\frac{\partial \omega}{\partial r}\ ,\ \frac{2}{y}\frac{\partial \omega}{\partial y}=\frac{2}{r}\frac{\partial \omega}{\partial r}$$

Now (ii), (iii), (iv) reduces to $-\mu y\left[\nabla^2\omega+\frac{2}{y}\frac{\partial \omega}{\partial y}+\frac{\sigma B_0^2}{\mu}\omega\right]=\frac{\partial p}{\partial x}$

$$-\mu x\left[\nabla^2\omega+\frac{2}{x}\frac{\partial \omega}{\partial x}+\frac{\sigma B_0^2}{\mu}\omega\right]=\frac{\partial p}{\partial y}\quad \&\quad \frac{\partial p}{\partial z}=0$$

$$Or\quad -\mu y\left[\frac{d^2\omega}{dr^2}+\frac{4}{r}\frac{d\omega}{dr}+\frac{\sigma B_0^2}{\mu}\omega\right]=\frac{\partial p}{\partial x}\qquad and\qquad \mu x\left[\frac{d^2\omega}{dr^2}+\frac{4}{r}\frac{d\omega}{dr}+\frac{\sigma B_0^2}{\mu}\omega\right]=\frac{\partial p}{\partial y}\quad \&\quad \frac{\partial p}{\partial z}=0$$

These are satisfied by p = constant, so $\frac{d^2\omega}{dr^2}+\frac{4}{r}\frac{d\omega}{dr}+\frac{\sigma B_0^2}{\mu}\omega=0$

$$let\ \frac{\sigma B_0^2}{\mu}=\ B^2$$

$$r\omega''(r)+4\omega'(r)+r\,B^2\omega(r)=0$$

Taking Laplace Transform

$$L\left\{r\omega''(r)\right\}+4L\left\{\omega'(r)\right\}+B^2L\left\{r\omega(r)\right\}=0$$

$$-\frac{d}{dp}\left[p^2L\left\{\omega(r)\right\}-p\omega(0)-\omega'(0)\right]+4\left\{pL\left\{\omega(r)\right\}-\omega(0)\right\}-B^2\frac{d}{dp}L\{\omega(r)\}=0$$

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$$-2pL\{\omega(r)\}-p^2\frac{d}{dp}L\{\omega(r)\}+A+4pL[\omega(r)]-4A-B^2\frac{d}{dp}L\{\omega(r)\}=0$$

$$-\left(p^2+B^2\right)\frac{d\bar{\omega}}{dp}+2p\bar{w}=3A \quad let \quad \bar{\omega}=L\{\omega(r)\}$$

$$\frac{d\bar{\omega}}{dp}-\frac{2p}{\left(p^2+B^2\right)}\bar{\omega}=-3A.\frac{1}{\left(p^2+B^2\right)}$$

$$I.F. = e^{-\int\frac{2p}{\left(p^2+B^2\right)}dp}=e^{-\log\left(p^2+B^2\right)}=\frac{1}{\left(p^2+B^2\right)}$$

$$\Rightarrow \bar{\omega}.\frac{1}{\left(p^2+B^2\right)}=-3A\int\frac{dp}{\left(p^2+B^2\right)}\frac{1}{\left(p^2+B^2\right)}+C=-3A\int\frac{1}{\left(p^2+B^2\right)^2}dp+C$$

$$=-3A\int\frac{\sec^2\theta d\theta}{B\sec^4\theta}+C \quad On\ putting \quad p=B\tan\theta \quad \Rightarrow dp=B\sec^2\theta\, d\theta$$

$$\Rightarrow\bar{\omega}\frac{1}{\left(p^2+B^2\right)}=-\frac{3A}{B^3}\int\cos^2\theta\, d\theta+C \quad =-\frac{3A}{2B^3}.\int\left(1+\cos 2\theta\right)d\theta+C$$

$$=-\frac{3A}{2B^3}.\left(\theta+\frac{\sin 2\theta}{2}\right)+C=-\frac{3A}{2B^3}\left[\tan^{-1}\frac{p}{B}+\frac{p}{\sqrt{B^2+p^2}}.\frac{B}{\sqrt{B^2+p^2}}\right]+C$$

$$=-\frac{3A}{2B^3}\left[\tan^{-1}\frac{p}{B}+.\frac{Bp}{\left(p^2+B^2\right)}\right]+C$$

$$\bar{\omega}=-\frac{3A}{2B^3}\left\{\left(p^2+B^2\right)\tan^{-1}\frac{p}{B}+Bp\right\}+C\left(p^2+B^2\right)$$

$$\therefore\ \omega(r)=-\frac{3A}{2B^3}\left[L^{-1}\left\{\left(p^2+B^2\right)\tan^{-1}\frac{p}{B}+BL^{-1}\{p\}\right\}\right]+\ CL^{-1}\left\{p^2+B^2\right\}$$

$$=-\frac{3A}{2B^3}L^{-1}\left\{\left(p^2+B^2\right)\tan^{-1}\frac{p}{B}\right\} \quad \because\ L^{-1}\left[p^n\right]=0 \quad Since\ n\ is\ a\ positive\ Integer$$

Now $\quad f(p)=\tan^{-1}\frac{p}{B} \quad \Rightarrow f'(p)=\frac{B}{B^2+p^2}$

$$\therefore\ L^{-1}\left[f^{'}(p)\right]=B\,L^{-1}\left[\frac{1}{\left(p^2+B^2\right)}\right]\quad = Sin\ r\,B$$

$$\Rightarrow\ -r\ L^{-1}[f(p)] = Sin\ r\,B \quad \Rightarrow\ L^{-1}[\tan^{-1}\frac{p}{B}] = -\frac{1}{r}Sin\ r\,B = f(r)$$

Now $\frac{d}{dr}\left[\frac{1}{r}Sin\ r\,B\right] = -\frac{1}{r^2}Sin\ r\,B + \frac{B}{r}Cos\ r\,B$

$$\frac{d^2}{dr^2}\left[\frac{1}{r}Sin\ r\,B\right] = \frac{2}{r^3}Sin\ r\,B - \frac{B}{r^2}Cos\ r\,B. - \frac{B}{r^2}.Cos\ r\,B - \frac{B^2}{r}.Sin\ r\,B$$

$$= \frac{2}{r^3}Sin\ r\,B - .\frac{2B}{r^2}Cos\ r\,B - \frac{B^2}{r}Sin\ r\,B$$

$$\therefore \omega(r) = \frac{3A}{2B^3}\left[\frac{2}{r^3}Sin\ r\,B - \frac{2B}{r^2}Cos\ r\,B - \frac{B^2}{r}Sin\ r\,B + \frac{B^2}{r}Sin\ r\,B\right]$$

$$\omega(r) = \frac{3A}{B^3}\left[\frac{1}{r^3}Sin\ r\,B - \frac{B}{r^2}Cos\ r\,B\right] = \frac{3A}{B^2r^3}\left[\frac{1}{B}\ Sin\ r\,B - r\ Cos\ r\,B\right]$$

Let the motion be produced by a solid sphere of radius a rotating with angular velocity ω' in a liquid at rest at infinity so that $\omega = 0$ at $r = \infty$ and $\omega = \omega'$ at $r = a$

$$\omega^{'} = \frac{3A}{a^3B^2}\left[\frac{1}{B}\ Sin\ a\,B - a\ Cos\ a\,B\right] \ \Rightarrow\ A = \frac{\omega^{'}a^3B^2}{3\left[\frac{1}{B}\ Sin\ a\,B - a\ Cos\ a\,B\right]}$$

$$\therefore\ \omega(r) = \frac{a^3\omega^{'}}{r^3}\ \frac{\left[Sin\ r\,B - r\,B\ Cos\ r\,B\right]}{\left[Sin\ a\,B - a\,B\ Cos\ a\,B\right]} \qquad ....(5)$$

Again if there exists an external fixed concentric spherical boundary of radius b i.e. (i) $\omega = 0$ at $r = b$ (ii) $\omega = \omega'$ at $r = a$

$$\frac{3A}{b^3B^2}\left[\frac{1}{B}Sin\ b\,B - b\ Cos\ b\,B\right] = 0 \quad \& \quad \omega' = \frac{3A}{a^3B^2}\left[\frac{1}{B}Sin\ a\,B - a\ Cos\ a\,B\right]$$

Adding both $\omega' = \frac{3A}{B^2}\left[\frac{1}{b^3B}\ Sin\ b\,B - \frac{1}{b^2}\ Cos\ b\,B + \frac{1}{a^3B}\ Sin\ a\,B - \frac{1}{a^2}\ Cos\ a\,B\right]$

$$A=\frac{\omega' B^2}{3\left[\frac{1}{b^3B}Sin\ bB-\frac{1}{b^2}Cos\ bB+\frac{1}{a^3B}Sin\ aB-\frac{1}{a^2}Cos\ aB\right]}$$

$$\therefore \omega(r)=\frac{\omega'\left[\frac{1}{B}Sin\ rB-r\ Cos\ rB\right]}{r^3\left[\frac{1}{b^3B}Sin\ bB-\frac{1}{b^2}\ Cos\ bB+\frac{1}{a^3B}\ Sin\ aB-\frac{1}{a^2}Cos\ aB\right]} \quad ....(6)$$

$Here\quad q_r=0,\quad q_\theta=0,\quad q_\phi=\omega r\ Sin\ \theta$

$$\frac{d\omega}{dr}=\frac{\omega'\left[\frac{-3}{B}\ Sin\ rB+3r\ Cos\ rB+Br^2Sin\ rB\right]}{r^4\left[\frac{1}{b^3B}\ Sin\ bB-\frac{1}{b^2}Cos\ bB+\frac{1}{a^3B}\ Sin\ aB-\frac{1}{a^2}\ Cos\ aB\right]}$$

$$\left(\frac{d\omega}{dr}\right)_{r=a}=\frac{\omega'\left[\frac{-3}{B}Sin\ aB+3a\ Cos\ aB+Ba^2\ Sin\ aB\right]}{a^4\left[\frac{1}{b^3B}\ Sin\ bB-\frac{1}{b^2}Cos\ bB+\frac{1}{a^3B}\ Sin\ aB-\frac{1}{a^2}\ Cos\ aB\right]}$$

The moment of p_ϕ is $p_\phi\ r\ Sin\theta$ where $p_\phi=\mu\ r\ Sin\theta\frac{d\omega}{dr}$ is the only non vanishing component of p. If N is the couple on the sphere of radius a, then

$$N=\int_0^\pi\left(P_\phi\ r\ Sin\theta\right)_{r=a}ds\quad=\int_0^\pi\mu\ a^2\ Sin^2\theta\left(\frac{d\omega}{dr}\right)_{r=a}.\ 2\pi a\ Sin\theta\ a\ .\ d\theta$$

$$N=2\pi\ \mu\ a^4\left(\frac{d\omega}{dr}\right)_{r=a}\int_0^\pi Sin^3\theta\ d\theta=4\pi\ \mu\ a^4\left(\frac{d\omega}{dr}\right)_{r=a}\int_0^{\pi/2}Sin^3\theta\ d\theta$$

$$=4\pi\mu\ a^4\left(\frac{d\omega}{dr}\right)_{r=a}\frac{\Gamma 2\ \Gamma 1/2}{2\ \Gamma 5/2}=2\pi\mu\ a^4\left(\frac{d\omega}{dr}\right)_{r=a}\frac{\sqrt{\pi}}{\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}}=\frac{8}{3}\pi\mu\ a^4\left(\frac{d\omega}{dr}\right)_{r=a} \quad ....(7)$$

But the rate dissipation of energy $=N\omega'\ =\frac{8}{3}\pi\mu\ a^4\left(\frac{d\omega}{dr}\right)_{r=a}.\omega'$

$$=\frac{8\pi\mu\,\omega'^2\left[\frac{-3}{B}Sin\;aB+3a\;Cos\;aB+Ba^2\;Sin\;aB\right]}{3\left[\frac{1}{b^3B}\;Sin\;bB-\frac{1}{b^2}\;Cos\;bB+\frac{1}{a^3B}\;Sin\;aB-\frac{1}{a^2}\;Cos\;aB\right]}$$

$$\lim_{b\to\infty}N=\frac{8\pi\mu\omega'\left[\frac{-3}{B}\;Sin\;aB+3a\;Cos\;aB+a^2B\;Sin\;aB\right]}{3\left[\frac{1}{a^3B}Sin\;aB-\frac{1}{a^2}Cos\;aB\right]}$$

$$=\frac{8\pi\mu a^3\omega'\left[\frac{-3}{B}\;Sin\;aB+3a\;Cos\;aB+a^2B\;Sin\,aB\right]}{3\left[\frac{1}{B}Sin\;aB-a\;Cos\;aB\right]}$$

$$=-8\pi\;\mu\;a^3\omega'+\frac{8\pi\;\mu\;a^5\omega'B}{3}\left[\frac{Sin\;aB}{\frac{1}{B}Sin\;aB-a\;Cos\;aB}\right]$$

For an infinite liquid outside a sphere of radius a, rate of dissipation of energy is

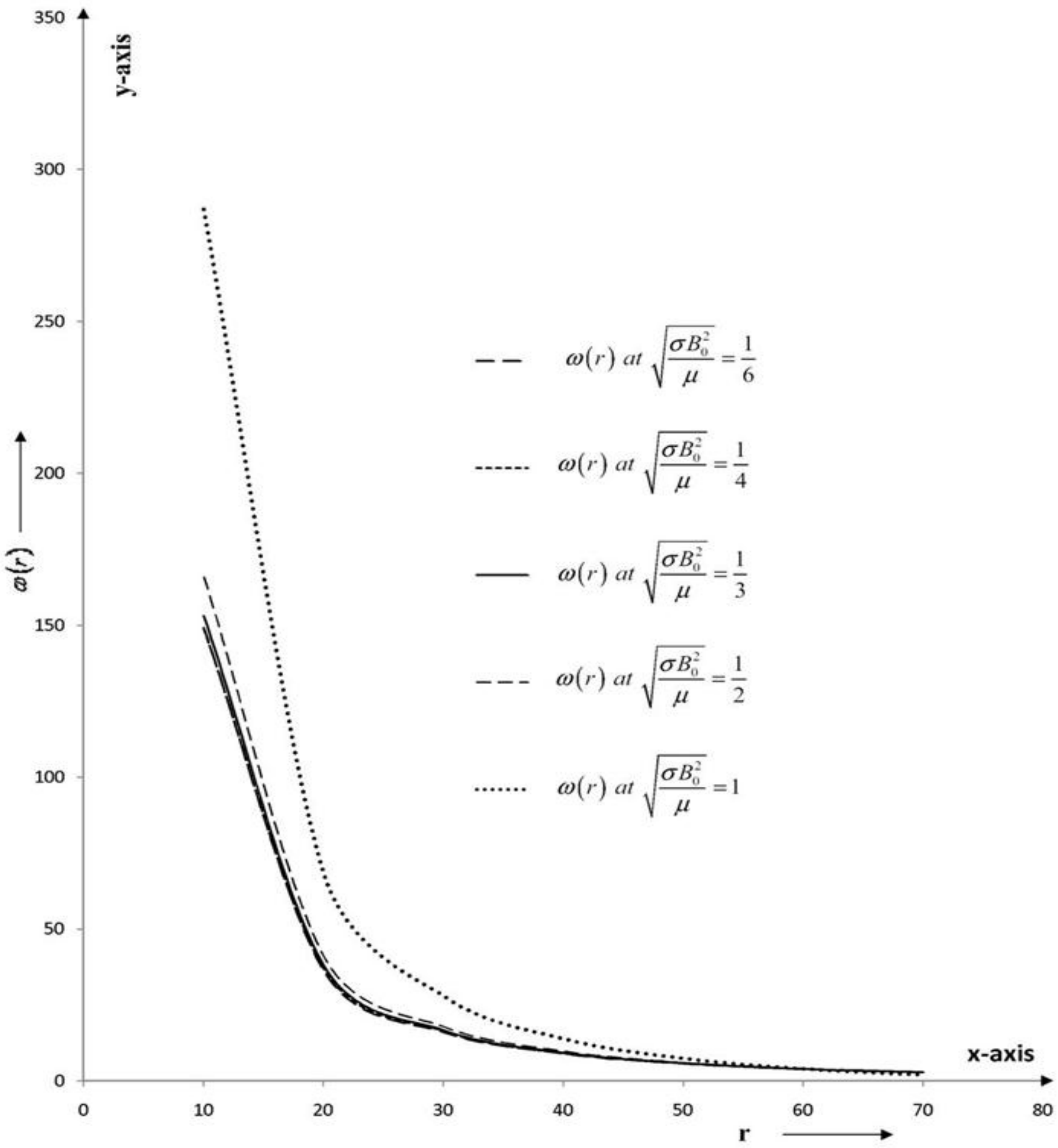
$$8\pi\;\mu\;a^3\omega'^2-\frac{8\pi\;\mu\;a^5\omega'B}{3}\left[\frac{Sin\;aB}{\frac{1}{B}Sin\;aB-a\;Cos\;aB}\right]\text{............(8)}$$

Table for Velocity: $Let\quad \omega'=4,\;\; a=60,\;\; are\,same\;and\quad B=\sqrt{\frac{\sigma B_0^2}{\mu}},\; r\;are\,change$

Table (1)

	r	10	20	30	40	50	60	70
$\sqrt{\frac{\sigma B_0^2}{\mu}}=\frac{1}{6}$	$\omega(r)$	**146.19**	**36.5**	**16.19**	**9.08**	**5.787**	**4**	**2.92**

$\sqrt{\frac{\sigma B_0^2}{\mu}} = \frac{1}{4}$	$\omega(r)$	**149**	**37.14**	**16.43**	**9.178**	**5.822**	**4**	**2.9**
$\sqrt{\frac{\sigma B_0^2}{\mu}} = \frac{1}{3}$	$\omega(r)$	**153.1**	**38.08**	**16.78**	**9.32**	**5.874**	**4**	**2.87**
$\sqrt{\frac{\sigma B_0^2}{\mu}} = \frac{1}{2}$	$\omega(r)$	**165.93**	**41**	**17.87**	**9.776**	**6.03**	**4**	**2.778**
$\sqrt{\frac{\sigma B_0^2}{\mu}} = 1$	$\omega(r)$	**286.9**	**68.40**	**27.99**	**13.9**	**7.44**	**4**	**1.989**



Velocity Profile

CONCLUSION AND DISCUSSION

By the graph of table (1) in the equation (5) between angular velocity and radius. It is clear that the angular velocity decreases in the interval $10 \le r \le 70$ at different value of $\sqrt{\frac{\sigma B_0^2}{\mu}}$ but the value of angular velocity is same $\{\omega(r) = 4\}$ at $r = 60$ for different values of $\sqrt{\frac{\sigma B_0^2}{\mu}}$. Again the value of angular velocity is less than the corresponding value of angular velocity in the interval $10 \le r \prec 60$ when the value of $\sqrt{\frac{\sigma B_0^2}{\mu}}$ increases , but the angular velocity is greater than the corresponding value of angular velocity in the interval $60 \prec r \le 70$ when the value of $\sqrt{\frac{\sigma B_0^2}{\mu}}$ increases. We have investigated the angular velocity given by the equation (6), couple on the sphere given by the equation (7) and rate of dissipation of energy given by the equation (8).

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