International Journal of Advanced Research in

Mathematics and Applications

Volume: 1 Issue: 1 08-Jan-2014,ISSN_NO: xxxx-xxx

Fuzzy Queueing Model Using DSW Algorithm

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ABSTRACT--- *This paper proposes a procedure to construct the membership functions of the performance measures in queueing systems where the interarrival time and service time are Fuzzy numbers. We propose a Fuzzy nature in FM/FM/1 queueing system with finite capacity and calling population are infinite. Approximate method of extension namely DSW(Dong, Shah & Wong) algorithm is used to define membership functions of the performance measures for the Queueing model FM/FM/1 in which the arrival rate and service rate are Fuzzy numbers. DSW algorithm is based on the α – cut representation of fuzzy sets in a standard interval analysis. The discussion of this paper is confined to systems with the Fuzzy variables. Numerical example shows the efficiency of the algorithm*.

Key words: Queueing theory, α - cut, Membership function, DSW algorithm.

1. INTRODUCTION

Queueing models have wider applications in service organization as well as manufacturing firms, in that various customers are serviced by various types of servers according to specific queue discipline [4] within the context of traditional queueing theory, the inter arrival times and service times are required to follow certain distributions. In usual practice the arrival rate, service rate are frequently described by linguistic terms such as fast, slow or moderate can be best described by the Fuzzy sets.

Li and Lee have derived analytical results for two fuzzy queueing system based on Zadeh's extension principle [10]. However, as commented by Negi and Lee [7] their approach is fairly complicated and is generally suitable for computational purposes.

Negi & Lee [7] propose two approaches the α - cut and two variable formulations, unfortunately their approach provides only possible numbers rather than intervals, in other words, the membership functions of the performance measures are not completely described.

This paper follows α – cut approach to decompose a fuzzy queue into a family of crisp queues. When α-varies the DSW(Dong, Shah and Wong) algorithm is used to describe the family of crisp queues. The solutions from the DSW derive the membership functions of the crisp queues.

To demonstrate the validity of the proposed approach the fuzzy queue FM/FM/1 where FM denotes the fuzzy in nature exponential time are exemplified.

Fuzzy queuing models have been described by such researchers like Li and Lee [6], Buckely [1], Negi ang Lee [7], Kao et al , Chen [2,3] are analyzed fuzzy queues using Zadeh's extension principle, Zadeh[9]. Kao et al constructed the membership functions of the system characteristic for fuzzy queues using parametric linear programming. Recently, Chen [2, 3] developed (FM/FM/1): (α/FCFS) and (FM/FM(k)/1):(α/FCFS). In practical, the queuing model, the input data arrival rate, service rate are uncertainly known. Uncertainty is resolved by using fuzzy set theory. Hence the classical queuing model will have more application if it is expanded using fuzzy models. We organize this paper as follows: Section 1, we give the literature survey of queueing models. In Section 2, we formulate the problem. In Section 3, Interval analysis arithmetic is defined. In Section 4, DSW algorithm is given. In Section 5, numerical example is iven. In Section 6, we conclude the problem.

2. PROBLEM FORMULATION

Consider a general queueing system with one server. The interarrival time αA and service time S are approximately known and are represented by the following fuzzy sets.

$$\tilde{A} = \left\{\left(a, \mu_{\tilde{A}}(a)\right) / a \in x\right\}$$

$$\tilde{S} = \left\{\left(s, \mu_{\tilde{S}}(s)\right) / s \in Y\right\}$$

Where x & y are the crisp universal sets of the inter arrival time & service time andμ (a)&μ (s) are the corresponding membership functions. The α - cuts or α- level sets of A & are the corresponding membership functions. The α - cuts or α- level sets of αA & S ~are

$$A(\alpha) = \left\{a \in x / \mu_{\tilde{A}}(a) \geq \alpha\right\},$$

$$S(\alpha) = \left\{s \in y / \mu_{\tilde{S}}(s) \geq \alpha\right\}$$

Where A(α) and S(α) are crisp sets using α - cuts, the inter arrival time and service time can be represented by different levels of confidence intervals. The membership function α p(A,s) α is constructed

$$\mu_{p(\tilde{A},s)} = \begin{cases} L(z), & z_1 \leq z \leq z_2 \\ 1, & z_2 \leq z \leq z_3 \\ R(z), & z_3 \leq z \leq z_4 \end{cases}$$

Where $z_1 \leq z_2 \leq z_3 \leq z_4$ and $L(z_1) = R(z_4) = 0$. Otherwise, since an attractive feature of the -cut approach is that all -cuts form a nested structure with respect to . The nested structures for expressions the relationship between ordinary sets & fuzzy sets, Chen[2005]. An approximate method of extension is propagating fuzziness for continuous valued mapping determined the membership functions for the output variables.

2.1 THE (FM/FM/1) : (/FCFS) QUEUES

This queue adopts a first-come first served discipline and consider an infinite source population where both the inter arrival time and the service time follow exponential distributions with ratio $\tilde{\lambda}$ and $\tilde{\mu}$ respectively, which are fuzzy variables rather than crisp values.The expected number of customer in the system

$$L_s = \frac{\lambda}{\mu - \lambda}$$

The average waiting time in the system

$$W_s = \frac{1}{\mu - \lambda}$$

The expected number of customers in the queue

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

The average waiting time of a customer in the queue

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

3. INTERVAL ANALYSIS ARITHMETIC

Let I1 and I2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

$$I_1 = [a, b],\ a \le b\ ;\ I_2 = [c, d],\ c \le d.$$

Define a general arithmetic property with the symbol *, where * = [+, -, ×, ÷] symbolically the operation.

$$I_1 * I_2 = [a, b] * [c, d]$$

represents another interval. The interval calculation depends on the magnitudes and signs of the element a, b, c, d.

$$[a,b]+[c,d]=[a+c,\ b+d]$$
$$[a,b]-[c,d]=[a-d,\ b-c]$$
$$[a,b]\bullet[c,d]=[\min(ac,ad,bc,bd),\ \max(ac,ad,bc,bd)]$$
$$[a,b]\div[c,d]=[a,\ b]\bullet\left[\frac{1}{d},\frac{1}{c}\right],\ \text{provided that}$$
$$0 \notin [c,\ d]$$
$$\alpha[a,b]=\begin{cases}[\alpha a,\alpha b] \text{ for } \alpha>0\\ [\alpha b,\alpha a] \text{ for } \alpha<0\end{cases}$$

4. DSW ALGORITHM

DSW(Dong, Shah and Wong) is one of the approximate methods make use of intervals at arious α - cut levels in defining membership functions. It was the full α-cut intervals in a tandard interval analysis. The DSW algorithm greatly simplifies manipulation of the extension rinciple for continuous valued fuzzy variables, such as fuzzy numbers defined on the real line. It revent abnormality in the output membership function due to application of the discrimination eaching on the fuzzy variable domain, it can prevent the widening of the resulting functional xpression by conventional interval analysis methods.

Any continuous membership function can be represented by a continuous sweep of α-cut nterm from α = 0 to α = 1. Suppose we have single input mapping given by y = f (x) that is to be extendedfor membership function for the selected α cut level.

International Journal of Advanced Research in

Mathematics and Applications

Volume: 1 Issue: 1 08-Jan-2014,ISSN_NO: xxxx-xxx

Step 4: Repeat steps 1 -3 for different values of α to complete a α cut representation of the solution.

5. NUMERICAL EXAMPLE

Consider a FM/FM/1 queue, where the both the arrival rate and service rate are fuzzy numbers represented by ʎ~=[1 2 3 4] andμ~= [15,16,17,18] . The interval of confidence at possibility level α as [1+α, 4–α] and [15+α, 18–α].

$$L_s = \frac{x}{y-x}, \quad W_s = \frac{1}{y-x}, \quad L_q = \frac{x^2}{y(y-x)},$$

$$W_q = \frac{x}{y(y-x)}$$

Where $x = [1+\alpha, 4-\alpha]$ & $y = [15+\alpha, 18-\alpha]$

TABLE: The α - cuts of Ls, Ws, Lq, Wq at α values

α	L_q	L_s	W_s	W_q
0	[0.0032, 0.1046]	[0.0588, 0.3636]	[0.0588, 0.0909]	[0.0032, 0.0261]
0.1	[0.0040, 0.0971]	[0.0655, 0.3482]	[0.0595, 0.0893]	[0.0036, 0.0247]
0.2	[0.0048, 0.0884]	[0.0723, 0.3333]	[0.0602, 0.0877]	[0.0040, 0.0233]
0.3	[0.0058, 0.0813]	[0.0793, 0.3190]	[0.0610, 0.0862]	[0.0044, 0.0220]
0.4	[0.0068, 0.0746]	[0.0864, 0.3051]	[0.0617, 0.0847]	[0.0048, 0.0207]
0.5	[0.0080, 0.0683]	[0.0938, 0.2917]	[0.0625, 0.0833]	[0.0052, 0.0195]
0.6	[0.0092, 0.0628]	[0.1031, 0.2787]	[0.0633, 0.0820]	[0.0058, 0.0185]
0.7	[0.0106, 0.0575]	[0.1090, 0.2661]	[0.0641, 0.0806]	[0.0062, 0.0175]
0.8	[0.0121, 0.0526]	[0.1169, 0.2540]	[0.0649, 0.0794]	[0.0067, 0.0164]
0.9	[0.0124, 0.0481]	[0.1250, 0.2422]	[0.0658, 0.0781]	[0.0072, 0.0155]
1	[0.0156, 0.0439]	[0.1333, 0.2380]	[0.0667, 0.0769]	[0.0078, 0.0146]

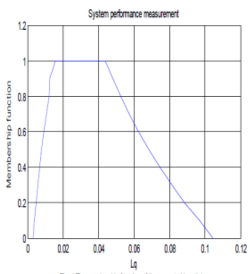

Fig. 1 The membership function of the expected length Lq

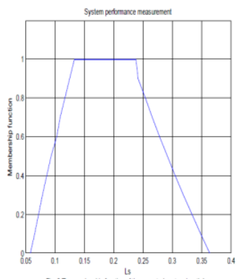

Fig. 2 The membership function of the expected system length Ls

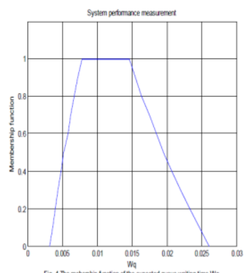

Fig. 4 The membership function of the expected queue waiting time Wq

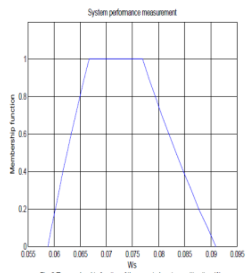

Fig. 3 The membership function of the expected system waiting time Ws

With the help of MATLAB 7.0.4, we perform α – cuts of arrival rate and service rate and fuzzy expected number of jobs in queue at eleven distinct α levels: 0, 0.1, 0.2, 0.3, …1. Crisp intervals for fuzzy expected number of jobs in queue at different possibilistic α levels are presented in Table. Similarly other performance measure such as expected number of jobs in the system (L_s), expected length of the queue (Lq), expected waiting time of job in queue (W_q) and expected waiting time of job in the system (W_s) also derived in Table.

The α – cut represent the possibility that these four performance measure will lie in the associated range. Specially, α = 0 the range, the performance measures could appear and for α = 1 the range, the performance measures are likely to be. For example, while these four performance measures are fuzzy, the most likely value of

the expected queue length L_q falls between

0.0156 and 0.0439 and its value is impossible to fall outside the range of 0.0032 and 0.1046; it is definitely possible that the expected waiting time in the system L_s falls between 0.1333 and 0.2380, and it will never fall below 0.0588 or exceed 0.3636. The above information will be very useful for designing a queueing system.

CONCLUSION

Fuzzy set theory has been applied to a number of queuing system to provide broader application in many fields. When the interarrival time and service time are fuzzy variables, according DSW algorithm, the performance measures such as the average system length, the average waiting time, etc., will be fuzzy. Numerical example shows the efficiency of the algorithm

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