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Determination of Optimal Reserves between Three Machines in Series

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ABSTRACT *- When a product is processed by two machines M_1 and M_2 in series, the breakdown of M_1 leads to the idle time of M_2 since the output of M_1 is the input for M_2. This is the problem of two machines in series. But when three machines are in series the stock or inventory of semi finished products between M_1 and M_2, as well as between M_2 and M_3 are required. The optimal level of inventory at two locations namely between M_1 and M_2 similarly between M_2 and M_3 is suggested. In this paper the optimal value of S_1 and S_2 namely the stock at two locations is found out by taking in to consideration the relevant costs of inventory holding and a shortage . Numerical illustrations are also provided.*

Key Words: Optimal Reserves, Semi Finished Products, Three Machines in Series.

1,Introduction:

In inventory control the size of the optimal inventory is determined taking in to consideration the relevant costs, the demand etc., The problem of determination of the optimal reserve of semi finished products between two machines in series has been attempted by Ramachandran and Sathiyamoorthy(1981).An extension of the model for the optimal reserve between two machines in series to the case of three machines in series has been discussed by Rajagopal and Sathiyamoorthy(2003).The determination of optimal reserves between three machines problem has been discussed by Venkatesan, Muthu and Sathyamoorthy(2011) by taking the breakdown duration of the first machine as the first order statistic and n^{th} order statistic respectively.The basic model has been discussed by Hansmaan (1962). In this paper an extension of the above model is discussed.

There are three machines in series. The output of machine M_1 is the input for the M_2 and the output of M_2 happens to be the input for M_3. The Optimal reserve inventory levels between machines M_1 and M_2 and similarly between the machines M_2 and M_3 is determined simultaneously taking in to consideration the relevant inventory holding cost as well as the cost

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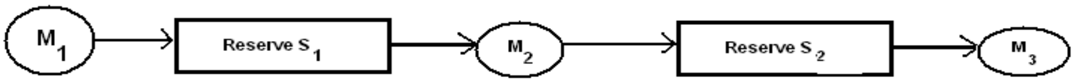
of shortages. In doing so it is assumed that the machine M_1 undergoes breakdown for a random duration and it is brought to the upstate after a random repair time . The consumption rate of the semi finished products by machine M_2 and also by machine M_3 are assumed to be random

variables. The following diagram gives an idea of the machines in series and the stocks in between the machines.



2,Assumptions

(i) There are three machines M_1, M_2 and M_3 in series.

(ii) The output of M_1 is the input for M_2 and the output of M_2 is the input for M_3.

(iii) The machine M_1 goes to the break down state for a random duration and if M_1 goes to the down state to keep M_2 and M_3 in upstate the reserve inventories between M_1 and M_2 and similarly M_2 and M_3 are maintained.

(iv) The consumption rates of machines M_2 and M_3 are random variables.

3,Notations

(i) h_1, h_2 = Inventory holding cost per unit of the semi finished products in S_1 and S_2 respectively per unit of time

(ii) d_1, d_2 = shortage cost or the idle time cost per unit time for machine M_2 and M_3 respectively

(iii) τ = down time duration of machine M_1 with p.d.f $g(\tau)$

(iv) r_1, r_2 = The consumption rate of machine M_2 and M_3 respectively per unit of time.

(v) μ_1, μ_2 = Average number of breakdowns for machine M_2 and M_3 due to shortage of input.

(vi) η = A random variable denoting the downtime of M_3 due to shortage.

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(vii) $\overline{F_2(.)}$ = denotes the reliability function of machine M_2 which implies that the down state of machine M_2 is only due to the failure of M_1 which forces M_2 to go to the idle state.

Under these assumptions the size of the optimal reserve inventory S_1 and S_2 is obtained.

The total expected cost due to inventory holding and shortages at the two locations in between M_1 and M_2 and similarly between M_2 and M_3

$$\left.\begin{aligned} E(C) = {} & \underbrace{h_1 \int_0^{s_1/r_1} r_1\left(\frac{s_1}{r_1}-\tau\right) g(\tau)\,d\tau}_{(A)} + \underbrace{\frac{d_1}{\mu_1}\int_{s_1/r_1}^{\infty}\left(\tau-\frac{s_1}{r_1}\right) g(\tau)\,d\tau}_{(B)} + \underbrace{h_2\int_0^{s_2/r_2} r_2\left(\frac{s_2}{r_2}-\tau\right) g(\tau)\,d\tau}_{(C)} \\ & + \underbrace{\frac{d_2}{\mu_1}\int_{\frac{s_1}{r_1}+\frac{s_2}{r_2}}^{\infty}\left(\tau-\frac{s_2}{r_2}-\frac{s_1}{r_1}\right) g(\tau)\,\overline{F_2(\tau)}\,d\tau}_{(D)} + \underbrace{\frac{d_2}{\mu_2}\int_{s_2/r_2}^{\infty}\left(\eta-\frac{s_2}{r_2}\right) h(\eta)\,d\eta}_{(E)} \end{aligned}\right\} ---(1)$$

let $g(\tau) = \lambda\, e^{-\lambda\tau}$

$$A = h_1 \int_0^{s_1/r_1} r_1\left(\frac{s_1}{r_1}-\tau\right) g(\tau)\,d\tau$$

Using Leibenitz rule , it is seen that

$$\frac{dA}{ds_1} = h_1\left[1-e^{-n\lambda s_1/r_1}\right] \qquad \text{..........(2)}$$

$$B = \int_{s_1/r_1}^{\infty}\left(\tau-\frac{s_1}{r_1}\right) g(\tau)\,d\tau$$

$$\frac{dB}{ds_1} = -\frac{d_1}{\mu_1 r_1}\left[e^{-n\lambda s_1/r_1}\right] \qquad \text{...........(3)}$$

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$$D = \int_{\frac{s_1}{r_1}+\frac{s_2}{r_2}}^{\infty} \left(\tau - \frac{s_2}{r_2} - \frac{s_1}{r_1}\right) g(\tau)\,\overline{F_2(\tau)}\,d\tau$$

Where $\overline{F_2(\tau)} = e^{-\theta\tau}$

$$\frac{dD}{ds_1} = -\frac{d_2 n\lambda}{\mu_1 r_1 (n\lambda+\theta)}\left[e^{-(n\lambda+\theta)\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)}\right] \qquad \ldots\ldots\ldots(4)$$

Substituting (2) , (3) and (4) in (1) we get

$$\frac{dE(C)}{ds_1} = 0 \Rightarrow$$

$$h_1\left[1-e^{-n\lambda^{s_1}/r_1}\right] - \frac{d_1}{\mu_1 r_1}\left[e^{-n\lambda^{s_1}/r_1}\right] - \frac{d_2 n\lambda}{\mu_1 r_1(n\lambda+\theta)}\left[e^{-(n\lambda+\theta)\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)}\right] = 0 \qquad \ldots\ldots\ldots(5)$$

Now to find $\frac{dE(C)}{ds_2} = 0 \Rightarrow$

$$C = h_2 \int_0^{s_2/r_2} r_2\left(\frac{s_2}{r_2} - \tau\right) g(\tau)\,d\tau$$

Using Leibnitz rule , it is seen that

$$\frac{dC}{ds_2} = h_2\left[1-e^{-n\lambda^{s_2}/r_2}\right]$$

and ………..(6)

$$E = \frac{d_2}{\mu_2}\int_{s_2/r_2}^{\infty}\left(\eta - \frac{s_2}{r_2}\right) h(\eta)\,d\eta$$

$$\frac{dE}{ds_2} = -\frac{d_2}{\mu_2 r_2}\left[e^{-\lambda\, n^{s_2}/r_2}\right]$$

……….(7)

$$D = \int_{\frac{s_1}{r_1}+\frac{s_2}{r_2}}^{\infty} \left(\tau - \frac{s_2}{r_2} - \frac{s_1}{r_1}\right) g(\tau)\,\overline{F_2(\tau)}\,d\tau$$

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$$= \int_{\frac{s_1}{r_1}+\frac{s_2}{r_2}}^{\infty} \left(\tau - \frac{s_2}{r_2} - \frac{s_1}{r_1} \right) n\,\lambda\, e^{-n\lambda\tau}\, e^{-\theta\tau}\, d\tau$$

$$\frac{dD}{ds_2} = -\frac{d_2 n\,\lambda}{\mu_1 r_2 (n\lambda + \theta)} \left[e^{-(n\lambda+\theta)\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)} \right] \qquad \ldots\ldots\ldots\ldots(8)$$

Substituting (6) , (7) and (8) in (1) we get

$$\frac{dE(C)}{ds_2} = 0 \Rightarrow$$

$$h_2 \left[1 - e^{-n\lambda^{s_2}/_{r_2}} \right] - \frac{d_2}{\mu_2 r_2} \left[e^{-\lambda\, n^{s_2}/_{r_2}} \right] - \frac{d_2 n\,\lambda}{\mu_1 r_2 (n\lambda+\theta)} \left[e^{-(n\lambda+\theta)\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)} \right] = 0 \qquad \ldots\ldots.(9)$$

Solving (5) and (9) we get

$$\frac{h_1}{r_2} - \frac{h_1}{r_2} e^{-n\lambda^{s_1}/_{r_1}} - \frac{d_1}{\mu_1 r_1 r_2} \left[e^{-n\lambda^{s_1}/_{r_1}} \right] - \frac{h_2}{r_1} = -\frac{h_2}{r_1} e^{-n\lambda^{s_2}/_{r_2}} - \frac{d_2}{\mu_2 r_1 r_2} \left[e^{-\lambda\, n^{s_2}/_{r_2}} \right] \qquad \ldots\ldots..(10)$$

$$-n\lambda \frac{s_1}{r_1} = \log \left[\frac{\mu_1 \left(e^{-n\lambda^{s_2}/_{r_2}} (h_2 \mu_2 r_2 + d_2) + \mu_2 (h_1 r_1 - h_2 r_2) \right)}{\mu_2 (d_1 + h_1 \mu_1 r_1)} \right]$$

$$\hat{S}_1 = \frac{r_1}{n\lambda} \log \left[\frac{\mu_2 (d_1 + h_1 \mu_1 r_1)}{\mu_1 \left(e^{-n\lambda^{s_2}/_{r_2}} (d_2 + h_2 \mu_2 r_2) + \mu_2 (h_1 r_1 - h_2 r_2) \right)} \right] \qquad \ldots\ldots.(11)$$

Again from (10)

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$$-n\lambda \frac{s_2}{r_2} = \log \left[\frac{\mu_2 \left(e^{-n\lambda s_1/r_1} (h_1\mu_1 r_1 + d_1) + \mu_1 (h_2 r_2 - h_1 r_1) \right)}{\mu_1 (h_2 \mu_2 r_2 + d_2)} \right]$$

$$\hat{S}_2 = \frac{r_2}{n\lambda} \log \left[\frac{\mu_1 (d_2 + h_2 \mu_2 r_2)}{\mu_2 \left(e^{-n\lambda s_1/r_1} (d_1 + h_1 \mu_1 r_1) + \mu_1 (h_2 r_2 - h_1 r_1) \right)} \right] \qquad \text{.........(12)}$$

4,Numerical Illustration:

$h_1 = 10,\ h_2 = 10,\ r_1 = 10,\ \mathrm{r}_2 = 12,\ \lambda = 1, \mathrm{d}_1 = 100, d_2 = 150,\ \mu_1 = 1.2, \mu_2 = 1.2, n = 10$

Substituting these values in (12) and increasing the values of S_1 the optimal S_2 values are obtained.

$\hat{S}_1$	$\hat{S}_2$
8	0.9070
9	0.9073
10	0.9076
11	0.9075
12	0.9077
13	0.9078
14	0.9077
15	0.9077
16	0.9077
17	0.9077

$\hat{S}_2 = 0.9077$ ....(13)

Substituting (13) in (11) we get

$\hat{S}_1 = 0.3947$ ....(14)

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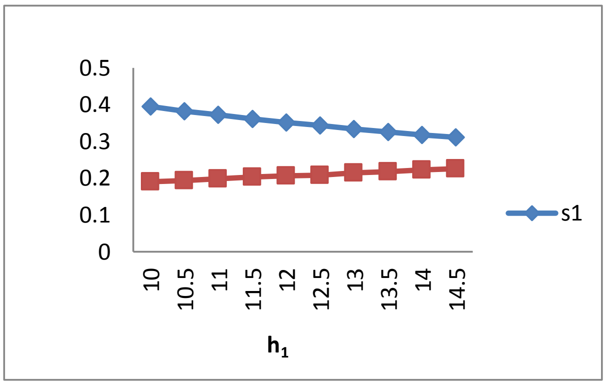
5,Numerical illustrations:

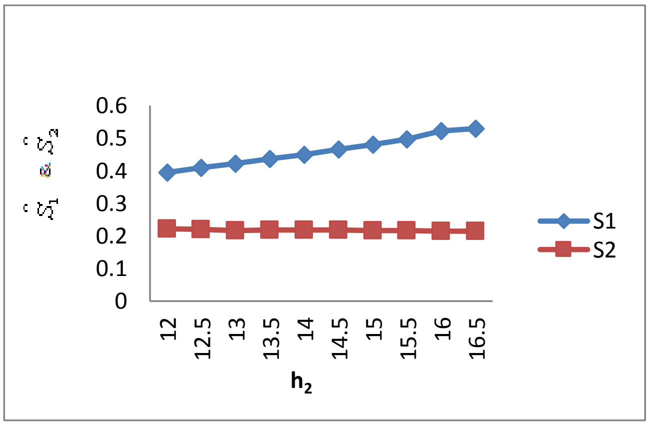
The changes in $\hat{S}_1$ and $\hat{S}_2$ due to the changes in h_1.

h_1	$\hat{S}_1$	$\hat{S}_2$
10	0.395	0.1908
10.5	0.3833	0.1946
11	0.3724	0.1985
11.5	0.362	0.2033
12	0.3523	0.2062
12.5	0.3431	0.2091
13	0.3343	0.2141
13.5	0.3261	0.2181
14	0.3182	0.2221
14.5	0.3108	0.2262



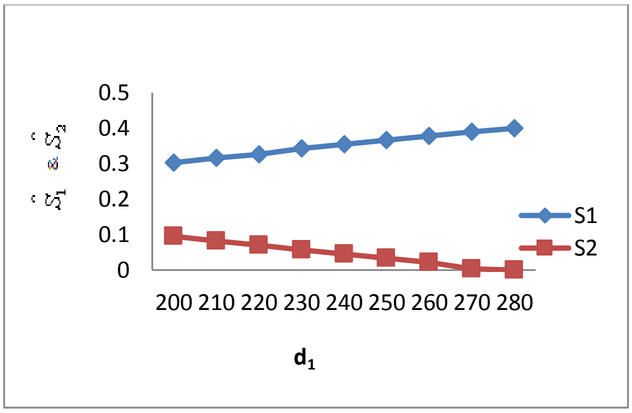
The changes in $\hat{S}_1$ and $\hat{S}_2$ due to the changes in h_2.

h_2	$\hat{S}_1$	$\hat{S}_2$
12	0.395	0.2209
12.5	0.4081	0.2201
13	0.4216	0.2162
13.5	0.4355	0.2184
14	0.4499	0.2177
14.5	0.4647	0.2169
15	0.4801	0.2162
15.5	0.4961	0.2155
16	0.5216	0.2148
16.5	0.5299	0.2142



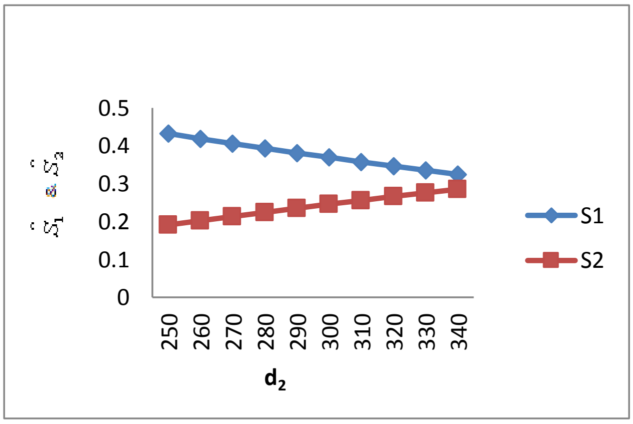
The changes of d_1 and the changes in $\hat{S}_1$ and $\hat{S}_2$

d_1	$\hat{S}_1$	$\hat{S}_2$
200	0.3033	0.0957
210	0.3167	0.0828
220	0.3267	0.0702
230	0.3423	0.0578
240	0.3545	0.0459
250	0.3664	0.0341
260	0.378	0.0226
270	0.3893	0.0032
280	0.4002	0.0003



The changes of d_2 and the changes in $\hat{S}_1$ and $\hat{S}_2$

d_2	$\hat{S}_1$	$\hat{S}_2$
250	0.4321	0.1907
260	0.4188	0.2022
270	0.4058	0.2133
280	0.3932	0.2243
290	0.381	0.235
300	0.3703	0.2455
310	0.3576	0.2558
320	0.3463	0.2658
330	0.3353	0.2757
340	0.3246	0.2855



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6, Conclusions

(i) As the inventory holding cost h_1 for the semi finished product between M_1 and M_2 increases, the value of S_1 decreases suggesting a smaller stock level . At the same time S_2 increases suggesting a higher inventory between M_2 and M_3 so as to bring down the idle time of M_3

(ii) If Inventory holding cost h_2 increases then a higher value of S_1 is suggested. At the same time the stock level should be smaller between M_1 and M_2.

(iii) If d_1 the shortage cost of inventory between M_1 and M_2 increases, a higher level of S_1 is suggested. But a smaller inventory between M_2 and M_3 is suggested.

(iv) As d_2, the shortage cost between M_2 and M_3 is on the increase a larger inventory between M_2 and M_3 is suggested so as to avoid the idle time M_3. But a smaller inventory between M_1 and M_2 is suggested.

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