International Journal of Advanced Research in
Electrical and Electronics Engineering
Volume: 2 Issue: 2 14-Feb-2014,ISSN_NO: 2321-4775

Performance Analysis of FIR Digital Filter Design Technique and Implementation

Mohd. Sayeeduddin Habeeb and Zeeshan Ahmad

Department of Electrical Engineering, King Khalid University,
Abha, Kingdom of Saudi Arabia

{mshabeeb, zayshan}@kku.edu.sa

.

ABSTRACT— *The purpose of this study is to design and analyze a Finite Impulse Response (FIR) filter to program a Pre-modulation filter for avionic application. In recent times more development is taking place in digital signal processing field (DSP). In DSP applications, more concentration is to reduce order of the filter to achieve high speed and low power, which results in less hardware requirements. In this paper various FIR filter design techniques are used and compared with optimal design techniques. Optimization technique which is used to design filter is McClellan-Parks which is based upon Remez exchange algorithm. Finally codes of different filter structures are implemented on a Field Programmable Gated Array (FPGA) and there hardware requirements are compared.*

Keywords— FIR filter, optimization, amplitude-frequency characterization, FPGA.

1, INTRODUCTION

The popularity of digital signal processing has been increased due to the declining cost of general purpose computers and due to applications to specific hardware. The need of digital filtering method is growing due to many applications like telephony and data communications is moving to preferring digital. To model this complex system, some simulation techniques is needed. The hardware simulation is more reliable than software because speed provided by hardware simulation is good, whereas building the hardware for different models is so costly and time consuming when some changes are made. Therefore, by using programming logic a middle ground may be found. Such systems offer software and some or all hardware flexibility [3] at the shorter implementation cost. A digital filter is exactly represented in the frequency domain.

In digital signal processing applications, digital filters are the most importantly used element. On the basis of impulse response, digital filter is classified into two types such as Infinite Impulse Response (IIR) filters, and FIR filters [1, 2]. FIR filter is preferred over IIR filter because it has linear phase and is easy to design. FIR filter is more stable and less sensitive to the length of filter.

FIR filters have no feedback so that there will be finite values. Differential equation of FIR filter is given as,

$$y[n] = b_0x[n] + b_1x[n-1] + \ldots\ldots\ldots\ldots + b_Nx[n-N] \tag{1}$$

where, $x[n]$ and $y[n]$ are input and output respectively, b_i are the filter coefficients and N is the order of filter (commonly referred to as taps). Convolution representation of equation (1) is given by,

$$y[n] = \sum_{i=1}^{N-1} b_i x[n-i] \quad (2)$$

Equation (2) is the one dimensional convolution between filter coefficient and input data. Therefore, output value of the filter is given by a weighted sum of the current and a finite number of previous values of the input. FIR filter avoids feedback and division, by using this advantage it is realizable in hardware. FIR filter has linear phase characteristic, which makes it ideal for most of the digital signal processing applications.

In this paper different filter design techniques are used for coefficients calculation. The most popular technique for the design is windowing techniques. A window design technique is easy but due to pass band ripples and frequency sampling, optimal filter design techniques are presented. In optimal design technique, coefficients are selected in a way to reduce the ripples in the pass band. The filter is then realized on FPGA. In realization part direct and transformed direct form is compared and based upon hardware better one is chosen.

This paper is structured as follows: Section 2 presents FIR filter impulse response and its structure. In section 3, different FIR filter design techniques are discussed. Section 4 presents the frequency response of the FIR filter. Section 5 discusses the VHDL implementation of the FIR filter. Finally we conclude the paper in Section 6.

2, IMPULSE RESPONSE OF FIR FILTER

From equation (2), by keeping $x[n] = \delta[n]$, impulse response $h[n]$ of the filter can be calculated. FIR filter impulse response becomes the set of coefficients $b_n$as follows,

$$h[n] = \sum_{k=0}^{N-1} b_k \delta[n-i] \quad (3)$$

So, $$h[n] = b_n \qquad \text{for } n = 0 \text{ to } N \quad (4)$$

Transfer function can find by taking z-transform of the impulse response as,

$$H(Z) = Z\{h[n]\}$$

$$H(Z) = \sum_{n=-\infty}^{\infty} h[z] z^{-n} \quad (5)$$

Using equation (4), equation (5) can also be written in the form,

$$H(Z) = \sum_{n=0}^{N} b_n z^{-n} \quad (6)$$

2.1 FIR filter structures

The most commonly used FIR filter implementation methods are direct-form and transpose-form whereas recursive implementation requires less computation steps for special filter. Occasionally sometimes lattice and cascade structures are used. Most straight forward method to realize FIR filter is direct form and it is most commonly used structure to implement [4]. This structure is called as non-recursive structure because there is no closed loop. So it is always possible to implemented FIR filter non-recursively because it can be implemented using the direct-form non-recursive structure. It is also possible to implement FIR filter recursively for some special case of filter coefficients.

Alternative to direct form is transposed direct form for realization of FIR filters. Transposed form is self-pipelined and it takes less area than direct form of realization. The delay can be added even in direct form or transpose form to make the design faster which result in mixed form. To maintain correctness of this design number of delays should be added with cut set algorithm [2].

In direct form extra pipeline register are added to reduce adders, delays and multipliers to achieve high throughput whereas in transpose direct form without adding any extra register high throughput can be achieved. In direct form symmetric FIR filter structures, we can implement symmetry condition to reduce numbers of multiplication by half i.e. reduce the number of multipliers from $N + 1\ to\ N/2 + 1$ [4].

3, FIR FILTER DESIGN

In designing FIR filter, most important parts are *approximation* and *realization*. Transfer function can be calculated in four steps after taking specification in approximation stage as,

- Usually in the frequency domain, desired or ideal response is chosen.
- Filter class is chosen which is allowed (e.g. the tap N for a FIR filter).
- Approximation quality is chosen.
- Lastly, best algorithm is selected which is used to find the transfer function

Implementation of the above transfer function in the form of circuit (blocks) or program (coding) is done by selecting the structure of filter, this stage is called as realization. Filter structure selection is important part in implementation on FPGA because of area and speed. Hardware implementation part in pre modulation cannot afford more area because of less space in on flight [4]. There are three types of FIR filter design techniques,

a) Windowing technique
b) Frequency sampling
c) Optimal design technique

We cannot achieve minimum order of filter with window design technique because it is a simple and convenient design technique for higher order filters. Rectangular, Blackman, Hamming, Hanning, Kaiser, Flat-top and Gaussian are some of the design techniques which are mostly used [5].

Frequency sampling design technique is the simplest and most direct technique if the desired frequency response is specified. In this technique desired frequency response can be obtain by sampling the frequency response which is provided by the previous method [4]. There are many optimal design techniques where we can specify pass and stop bands. Some of these techniques are equiripple and least square methods. Most important type of

International Journal of Advanced Research in

Electrical and Electronics Engineering

Volume: 2 Issue: 2 14-Feb-2014,ISSN_NO: 2321-4775

optimal design technique is *Parks – McClellan algorithm* [6]. In this paper this algorithm is still optimized such that pass band error is reduced.

3.1 Window Design

This is also called as Fourier transform method and is widely used designing method. Causal and linear phase FIR filters can be obtained by truncating infinite impulse sequence; infinite length impulse response cannot be realized. Finite length filter can be raised by completely truncating all values outside of a certain range. $H_d(e^{jw})$ represents ideal frequency response of filter and it is periodic in frequency and can be represented in Fourier series. Let $h_d(n)$ is the impulse response of filter and it is given by,

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{jw})\, e^{jw} dw \tag{7}$$

$$H_d(e^{jw}) = \sum_{n=-\infty}^{\infty} h_d\,(n) e^{-jwn} \tag{8}$$

FIR filter can be obtained by truncating (approximating) this infinite series at $\mathrm{n} = \pm\left(\frac{\mathrm{N}-1}{2}\right)$, where N is length of filter [7]. Direct truncation or non-uniform convergence results in *Gibb's phenomenon*. It results into the overshoot and ripples in the spectrum. By multiplying infinite impulse response with finite window $w(n)$ given in equation (9), this can be result to Gibb's phenomenon.

$$w(n) = w(-n) = \begin{cases} \neq 0 & \text{for } |n| \leq \left(\frac{N-1}{2}\right) \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

3.1.1 Rectangular Window

This is the simplest window design method, but at the stop band this window provides worst performance. It is represented by Fourier transform of the unit pulse of sinc function,

$$w(n) = \begin{cases} 1 & 0 \leq n \leq M-1 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$= e^{-jw(N-1/2)} \frac{\sin(wN/2)}{\sin w/2} \tag{11}$$

$$W(e^{jw}) = \sum_{n=0}^{N-1} e^{-jwn} = \frac{1-e^{-jwn}}{1-e^{-jw}} \tag{12}$$

3.1.2 Bartlett Window

This is simply known as triangular window. Sharp transition of rectangular window from 0 to 1 (1 to 0) results to Gibbs's phenomenon; so here frequency response will approaches zero smoothly in the form of a triangle [4]. Triangular function produces smooth magnitude repose in pass band and stop band. With this method a transition region is more and attenuation is less in stop band [7], which is its disadvantage. Because of this problem triangular window is not used.

Volume: 2 Issue: 2 14-Feb-2014,ISSN_NO: 2321-4775

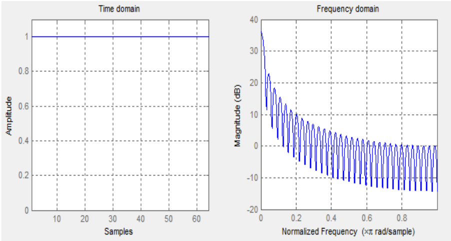

Fig. 1 Rectangular Window for N=64

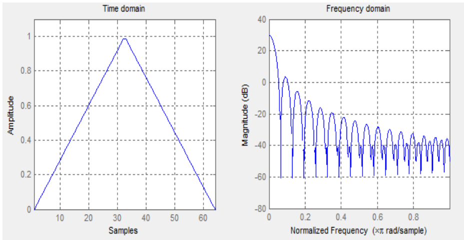

Fig. 2 Bartlett Window for N=64

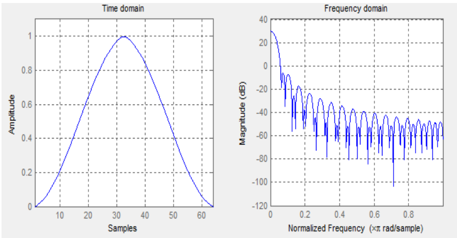

Fig. 3 Hanning Window for N=64

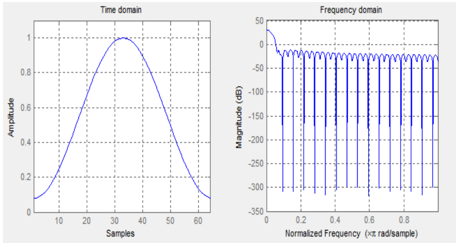

Fig. 4 Hamming Window for N=64

3.1.3 Hanning Window

This is also called as raised cosine window. Here transition region of filter is double because the width of main loab in Hanning window is twice of the rectangular window. First side loab for Hanning window is one tenth of rectangular window, because of this there are less ripples in stop band and pass band regions [7].

3.1.4 Hamming Window

This window is similar to that of Hanning window except that it has small amount of discontinuities at the boundaries.

3.1.5 Blackman Window

It is similar to hamming and hanning windows but it has an additional cosine term for the reduction of ripples in pass band and stop band, and improves the width of main loab. Suitable window type can be selected and widow size is based upon given transition width and minimum stop band attenuation of the desired filter information as described in Table.1.

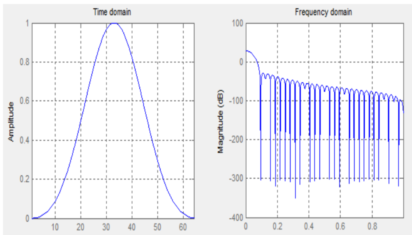

Fig. 5 Blackman Window for N=64

3.1.6 Kaiser window

Fixable family of window is defined by Kaiser and equation is represented by

$$w(n) = \begin{cases} I_0\left(\beta(1-((n-\alpha)/\alpha)^2)^{\frac{1}{2}}\right)/I_0(\beta) & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases} \qquad (13)$$

Where $\alpha = (N-1)/2$ and I_0 = the zeroth order modified Bessel Function.

Table.1 Summary of commonly used window characteristics

Window Name	Transition Width		Min. Stop band Attenuation
	Approximate	Exact Values	
Rectangular	$\frac{4\pi}{M}$	$\frac{1.8\pi}{M}$	21 dB
Bartlett	$\frac{8\pi}{M}$	$\frac{6.1\pi}{M}$	25 dB
Hanning	$\frac{8\pi}{M}$	$\frac{6.2\pi}{M}$	44 dB
Hamming	$\frac{8\pi}{M}$	$\frac{6.6\pi}{M}$	53 dB
Blackman	$\frac{12\pi}{M}$	$\frac{11\pi}{M}$	74 dB

3.2 Summary of windows

FIR filter designed by Bartlett window will reduces the peak amplitude or overshoot of main loab but widens transition region. For getting smooth and better truncations of ideal frequency responses, the function like Hamming, Hanning and Blackman are more complicated. Kaiser provides better window results because parameter β can be used to compromise between transition region width spreading reduction of peak amplitude (overshoot) and transition region width spreading.

Windowing method is simple and easy to use compare to that of other technique because filter coefficient can be easily calculated from well-defined equations. It also has some problems using windowing method as follow.

- This method is applicable if $H_d(w)$ is absolutely integrable then calculating of $h_d(n)$ is possible. If $H_d(w)$ is complicated then calculation of $h_d(n)$ will become difficult.
- Because of discontinuity in frequency generally pass band edges cannot be defined exactly. So these filters are not flexible.
- It is used in speech processing and image processing applications. Because basically windows design techniques used to design filter like low pass, high pass, band pass etc.

3.3 Frequency Sampling Design Technique

Frequency sampling design technique is simplest and most direct technique if the desired frequency response is specified. In this technique [1, 8, 9] desired frequency response can be obtain by sampling, frequency response which is provided by the previous method. Sampling is done at the particular set of equally spaced frequency to obtain N number of samples. Frequency response $H_d(w)$ is sampled at N point which gives us N-point Discrete Fourier Transform (DFT) of $H_d(2\pi nk/N)$. Coefficient of this filter can be calculate by using Inverse DFT as,

$$h(n) = \frac{1}{N}\sum_{n=0}^{N-1} H(k)\, e^{j(2\pi n/N)k} \quad (14)$$

3.4 Optimal Filter Design Methods for digital FIR filter design

For equiripple FIR filters design, first algorithm was developed by Herrmann and others [7]. In this algorithm $M, \delta s, \delta p$ are fixed and ωs and ωp are variable. In the algorithm by Parks and McClellan and others ωs , $\omega p, M$ and ratio $\delta p/\delta s$, are fixed and δs *or* δp, are variable. Stop band and pass band of equiripple design is equally weighted and has linear phase characteristics the filter parameters are $M, \delta s, \delta p, \omega s$, ωp and it is not possible to specify these parameters independently.

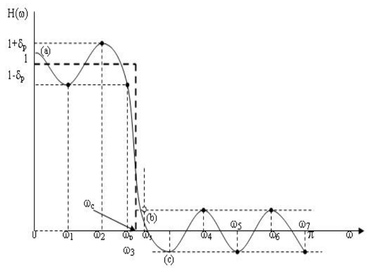

Fig. 6 Equiripple approximation of a low pass [7]

Based upon these parameters two design algorithms are developed in which some of the parameters are fixed and some parameters are adjusted optimally by interpolation [9]. For designing of FIR filters two different approaches were developed.

4, FREQUENCY RESPONSE OF FIR FILTERS

Frequency response of FIR filter using different window design technique is given in Fig. 7- Fig. 10 for the filter specifications given in Table. 2.

Table. 2 FIR filter specifications

Items	Specifications
FIR order	6
FIR attenuation at –3db freq	-0.05db
Structure	Transposed Direct form
Freq response	Linear phase
Design algorithm	Equiripple technique
Sampling freq	8∗bit rate
Cut-off freq	1.4∗bit rate

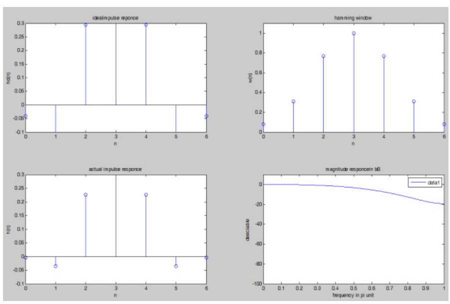

Fig. 7 Frequency Response of Hamming Window

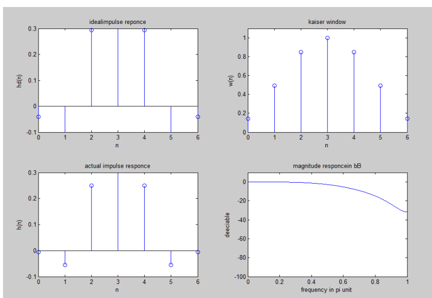

Fig. 8 Frequency response of Kaiser Window

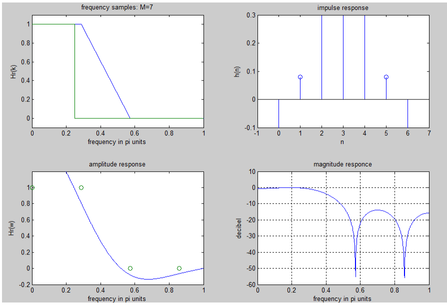

Fig. 9 Frequency response of frequency sampling

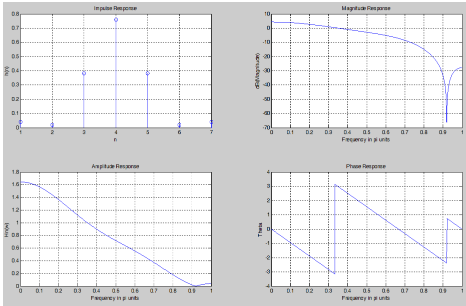

Fig. 10 Frequency response of Optimal FIR filter

5. VHDL IMPLEMENTATION

Length of FIR filter is 6 and there are seven numbers of coefficients, FIR filter coefficient length are not so long from Table. 3. FIR filter hardware needs very small allocation area on chip as vertex-4 board has around 30,720 gates. From the simulation report it will be clear that how much hardware does the direct form of filter requires. For the Direct form FIR filter structure, the filter response ends in finite amount of time because impulse response is finite for FIR filter.

Coefficient of filter must be in integer format, which is required for applying to hardware circuitry. So multiplying the coefficient by 215, 16 bit integer values is calculated. New coefficient values are given in Table. 3. FIR filter is implemented in VHDL. Simulation results for direct form FIR filter with input sequence as filter_in and corresponding output value is 24917 and minimum value is 668.

Table. 3 Calculated Coefficients.

h(0)	1298	***h(4)***	12501
h(1)	668	***h(5)***	668
h(2)	12501	***h(6)***	1298
h(3)	24917		

5.1 Direct form FIR filter

HDL Synthesis Report

Macro Statistics

# Multipliers	: 7
16x16-bit multiplier	: 7
# Adders/Subtractors	: 6
32-bit adder	: 1
33-bit adder	: 5
# Registers	: 8
16-bit register	: 7
32-bit register	: 1

Advanced HDL Synthesis Report

Macro Statistics

# Multipliers	: 7
16x16-bit multiplier	: 7
# Adders/Subtractors	: 6
32-bit adder	: 6
# Registers	: 112
Flip-Flops	: 112

5.1.1 VHDL Simulation Results

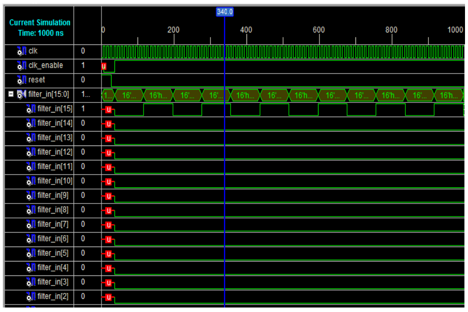

Fig. 11 HDL Input to direct form FIR filter

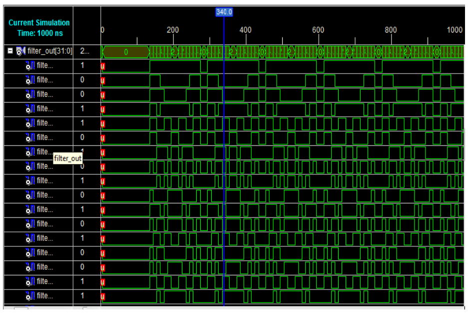

Fig. 12 Output of direct form FIR filter

5.2 Transform Direct form FIR filter

HDL Synthesis Report

Macro Statistics	
# Multipliers	: 4
16x16-bit multiplier	: 4
# Adders/Subtractors	: 6
33-bit adder	: 6
# Registers	: 8
16-bit register	: 1
32-bit register	: 7

Advanced HDL Synthesis Report

Macro Statistics	
# Multipliers	: 4
16x16-bit registered multiplier	: 4
# Adders/Subtractors	: 6
32-bit adder	: 6
# Registers	: 223
Flip-Flops	: 223

From the above synthesis report it is clear that FIR filter with the direct form implementation requires 7 multipliers and 7 adders to perform filtering, comparing this transformed direct form implementation. With the transformed direct form, number of multipliers used are 4 only.

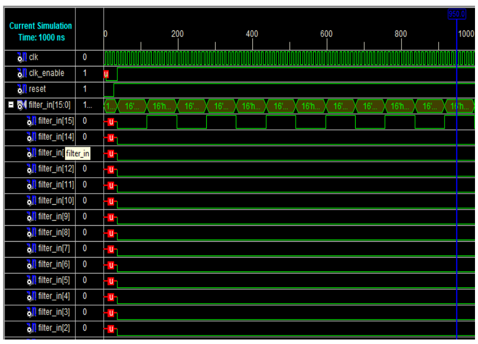

Fig. 13 Input to transformed direct form FIR filter

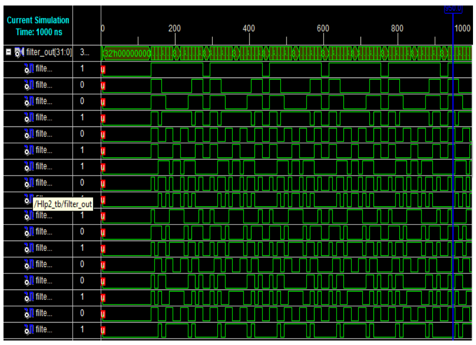

Fig. 14 Output of transformed direct form FIR filter

5.2.1 VHDL Simulation results

Simulation results for transposed direct form are shown in Fig. 13 and 14. Input to filter is filter_in and clock as clk and output as filter_out.

6, CONCLUSION

FIR filters are most commonly used in DSP applications because it can be programmed. In this paper, FIR filter design techniques and its structures has been discussed where each technique has its own advantages and disadvantages like window design technique and frequency design technique is quite easy to design and implement. They have some draw-backs like they do not have band specification like pass band ripple and stop band ripple user has to accept whatever they got from design. So concept of optimal design technique is implemented in which user can define pass band ripple and stop band ripples and with this optimal design technique errors can reduced. Parks-McClellan algorithm is dominant method of designing optimal FIR filter. Then VHDL implementation is performed and from the synthesis report of direct form and transposed direct form it is observed that transposed form require less hardware compare with direct form representation, so for pre modulation this FIR filter can be used to avoid major change in flight and hardware can be reduced in future application. This approach gives a better performance than the common filter structures in terms of speed of operation, cost, and power consumption.

REFERENCES

[1]. J.G.Proakis and D.G.Manolakis, "*Digital Signal Processing Principles, Algorithms and Applications*" third edition, Prentice-Hall, 2002.

[2]. John G. Proakis and Dimitris G. Manolakis, "*Signal Processing principles. Algorithms, and Application*", 3rd edition, Prentice-Hall of India privet limited, New Delhi, India, 2006

[3]. R. E. Bogner and A. G. Constantinides, "*Introduction to Digital Filtering*", John Wiley & Sons, Ltd., 1975.

[4]. Vinay K. Ingle and John G. Proakis, "*Digital signal Processing using MATLAB*", International Student Edition, Vikas Publishing House, 2003

[5]. S. M. Shamsul Alam , Md. Tariq Hassan, "*Performance Analysis of FIR Filter Design by Using Optimal, Blackman Window and Frequency Sampling Methods*", International Journal of Electrical & Computer Sciences IJECS-IJENS Vol: 10 No: 01

[6]. A. V. Oppenheim and R. W. Schafer, "*Discrete-Time Signal Processing*", Prentice-Hall, Englewood Cliffs, New Jersey, 1989, pp. 236, and 441

[7]. P. Ramesh Babu, "*Digital signal processing*", 3rd edition, Scitech Publication (India) Pvt. Ltd.,2007

[8]. T.W. Parks and C.S. Burrus, "*Digital Filter Design*",New York: Wiley, 1987

[9]. L.R. Rabiner and B. Gold, "*Theory and Applications of Digital Signal Processing*", New Jersey: Prentice-Hall, 1975