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VIBRATION ANALYSIS OF FUNCTIONALLY GRADED BEAM BY USING FINITE ELEMENT METHOD

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Abstract-

The dynamic characteristics of functionally graded beam in which material properties changes either in an axial direction or along the thickness of beam following a simple power law are studied. The system of equations of motion is consequent by using the Lagrange's principle under the assumptions of the Euler–Bernoulli beam theory. Resulting system of ordinary differential equations of free vibration analysis are solved using an analytical method. The finite element method is employed to discretize the model and obtain a numerical approximation of the motion equation. The model has verified with the previously published works and found a good conformity with them. Numerical results are presented in tabular form to figure out the effects of different material distribution and boundary conditions on the dynamic characteristics of the beam.

Introduction

Functionally graded material (FGM) is an intentionally developed material to fulfill positive function. First attempt was made in 1984, in Sendai area of Japan, to manufacture FGM. They are heterogeneous, anisotropic materials made up of ceramic on one side and metal on the other. Hence the material properties change gradually with location within the FGM. The ceramic, low thermal conductivity material, generally facing high temperature side guards the metallic surface from corrosion and thermal failure, whereas the metallic part gives strength and stiffness to the structure generally presents on low temperature side. The benefit of using FGM is that they are able to withstand high temperature gradient surroundings while maintaining their structural reliability. The material properties are modeled varying according to a power law.

FGM is different from composites wherein the volume fraction of the presence is uniform throughout the composite. The closest analogies of FGMs are covered composites, but the latter possess distinctive interfaces across which properties change abruptly. It possesses properties that vary gradually and continuously with respect to the longitudinal or transverse directions in order to achieve a necessary function. The composition is varied from a ceramic rich surface to metal rich surface with a desired variation of the volume fraction of the two materials in between two surfaces can be easily achieved . Initially, FGMs were designed as thermal barrier materials for aerospace application and fusion reactors. Later on, FGM are developed for military, manufacturing industry, biomedical application, automotive, semiconductor industry and general structural element in thermal environments.

The literature on the analytical free vibration analysis of FG beams is limited certain articles. Chakrabortyetal developed a new beam element to study the thermo elastic behavior of FG beam structures considering shear deformation. S.A. Sina, H.M. Navazi , H. Haddadpour studied An analytical method for free vibration analysis of

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functionally graded beams. B.V. Sankar developed an elasticity solution for functionally graded beams. In his solution, an exponential function was assumed to describe the beam's stiffness variation through thickness. However, Sankar's model was limited to long beams with slowly varying transverse loadings. Rajesh K. Bhangale, N. Ganesan worked on Thermo elastic buckling and vibration behaviour of a functionally graded sandwich beam with constrained viscoelastic core. Failure mode survey of slot in beams with functionally graded core analysis was carried out by Antonio F. A Vila. Free vibration characteristics of a functionally graded beam by finite element method were studied by Amal E. Alshorbagy, M.A. Eltaher, and F.F. Mahmoud.

The mathematical model

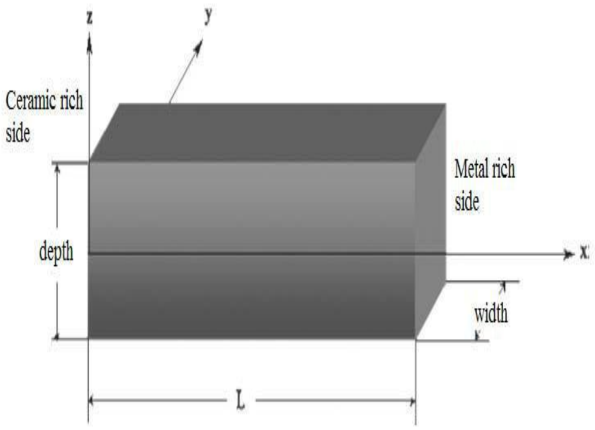

Fig A typical FG beam

The FG beam having length L, width b and depth or thickness h is shown in the figure above.

The material properties –

In the present study, the material properties, like density ρ, modulus of elasticity E etc. are assumed to be varying axially i.e. along X direction according to a simple power law given by

$P(Z)=(P_M-P_C)(z/h+1/2)_n +P_C$

where Pc and Pm are the corresponding material properties of the ceramic and metal, and n is the volume fraction exponent, which have values greater than or equal to zero. We have considered material properties are varying axially

Lagranges principle

Beam element.

After following basic procedure of lagranges principle we get following shape functions

$N1=(1-\xi)$
$N2= \xi$
$N3=(1- 3\xi_2 +2 \xi_3)$
$N4=a \xi(1-2 \xi+ \xi_2)$
$N5= \xi_2(3-2 \xi)$
$N6=a \xi_2(\xi-1)$

Mass matrix=

[■(■(∫↓0^1 【(ρ】 ξN1N1dX&0&0&∫↓0^1 【(ρ】 ξN1N2)dX&0&0@0&∫↓0^1 【(ρ】 ξN3N4)dX&∫↓0^1 【(ρ】 ξN3N4)dX&0&∫↓0^1 【(ρ】 ξN3N5)dX&∫↓0^1 【(ρ】 ξN3N6)dX@0&

Stiffness matrix=E_x A/a

[■(■(∫↓0^1 (N1'N1'dX&0&0&∫↓0^1 (N1' N2')dX&0&0@0&∫↓0^1 (N3"N3")dX&∫↓0^1 (N3"N4")d

Results

First 5 natural frequencies (in rad/sec) for 3 different boundary conditions is tabulated below

Material property exponent k=1, length L = 500mm, width b = 25mm, depth h = 25mm

Simply Supported Beam	Clamped Clamped Beam	Cantilever Beam
105.7	238.1	37.3
424.0	660.1	234.8
955.2	1298.3	660.0
1700.6	2152.9	1249.2
2664.4	3229.8	2149.2

Conclusion

It is seen that, natural frequency for clamped – clamped beam is the highest. For cantilever beam it is the lowest.

References

1) B.V. Sankar, An elasticity solution for functionally graded beams, Compos. Sci. Technol. 61 (2001) 689–696.

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